A Life Worth Mentioning

Conway's Game of Life and Other Notable Life-like Rules

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Cellular automata (CA) have been lauded for their ability to generate complex global patterns from simple and local rules. The late English mathematician, John Horton Conway, developed his illustrious Game of Life (Life) CA in 1970 and it remains the quintessential CA construction—capable of producing a myriad of complex dynamic patterns and computational universality. Life and several other Life-like rules are usually classified in the same group of aesthetically and dynamically interesting CA rules characterized by their complex behaviors. However, a rigorous quantitative comparison among similarly classified Life-like rules has not been fully established. We use a modified conditional entropy measure, which considers an information-theoretic distance from randomness, as a measure of complexity (Δ) for two dimensional (2D) CA patterns. We use this measure to show that Life is likely optimal in that it naturally aims to minimize density and maximize information content while being the most parsimonious Life-like rule with this feature. That is, Life is capable of maintaining a consistent amount of long-lasting complexity with the least number of conditions compared to other Life-like rules. We also show that the complexity of higher density Life-like rules, which themselves contain the Life rule (i.e., B3/S23), form a distinct density-complexity relationship whereby (B356/S23) is proposed as an optimal complexity candidate (Fig.1). Generally, our results support that Life functions as the basic ingredient for cultivating the balance between structure and randomness thereby maintaining complexity in 2D CA over many evolutions. This work highlights the genius of John Horton Conway and his Game of Life—a testament to the timeless marvel which is certainly a *life worth mentioning*.
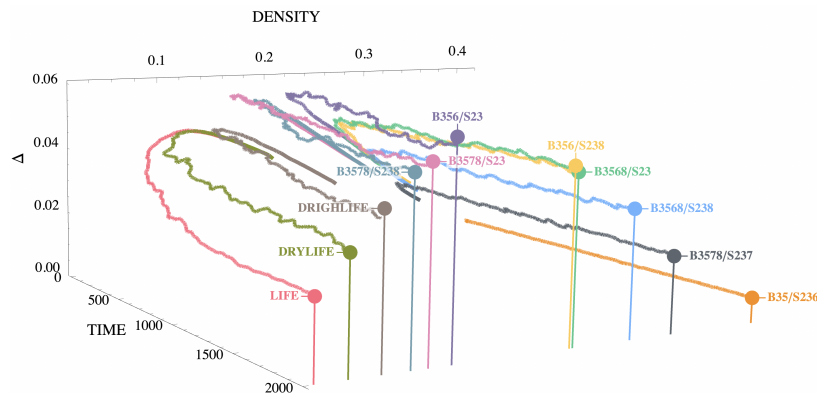
Figure 1: Density and complexity (Δ) relationship is revealed among Life-like rules that contain Life showing that Game of Life is fundamental and complexity is optimized with B356/S23.